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K-models and Type IIB Superstring Backgrounds

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Abstract

A family of type IIB superstring backgrounds involving Ramond-Ramond fields are obtained in ten dimensions starting from a K-model through a generalization of our recent results. The unbroken global $SL(2, R)$ symmetry of the type IIB equations of motion are implemented in this context as a solution generating transformation. A geometrical analysis, based on the tensor structure of the higher order α' terms in the equations of motion, is employed to show that these backgrounds are exact.

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1. Introduction

Recent advances in string theory has provided a glimpse of their underlying non-perturbative structure. There now seems to be strong indication that the several distinct string theories all arise from a yet unknown fundamental theory in eleven dimensions, which has been christened as M-theory [1]. The central guiding principle behind this unification has been the duality symmetries [2, 3, 4, 5]. They include the T-duality [2] which is perturbative in the genus expansion and the conjectured non-perturbative S-duality [3, 4, 5]. The latter relates electrical charged perturbative states to magnetic charged solitons and weak to strong coupling regimes. As an example, we have the type IIB string theory in ten dimensions [6, 7]. These possess a global $SL(2, R)$ symmetry of the classical effective field theory which is broken to the discrete $SL(2, Z)$ S-duality group in the quantum theory which relates NS-NS to the R-R sector.

In the sigma model approach to string theory [8], one considers the propagation of a string in the background of its massless excitations. The evolution is described by a two dimensional sigma model in which the background fields appear as couplings. Conformal invariance then requires the corresponding perturbative beta functions for these couplings to vanish, leading to the background field equations of motion. The higher order terms [8] in these equations provide the stringy corrections to the tree level theory and may be computed perturbatively. However for the type IIB string background, this remains a non-trivial exercise owing to the presence of R-R fields which arise from the solitonic sector [9]. These couple to the spin fields on the worldsheet and makes the corresponding perturbation theory intractable at higher orders [9, 10]. There exists however, a large class of string backgrounds for the bosonic and heterotic cases, where the higher order contributions are identically zero [11, 12, 13, 14, 15]. Hence the tree level equations of motion are exact to all orders in the sigma model coupling α' . Amongst these are the class of backgrounds with a covariantly constant null Killing vector which are known as K-models [11, 13]. The

simplest example in this class are the plane wave string backgrounds. For the bosonic and heterotic versions of these, it has been shown that they are exact (in α') through a purely geometrical analysis based on the existence of the covariantly constant null killing vector [12, 16, 17, 18].

Previously, in collaboration with Kar [19], we have shown that starting from such a plane wave background embedded trivially in a type IIB string theory (i.e with vanishing R-R fields), it is possible to generate a non-trivial type IIB background with R-R fields in ten dimensions. We further showed, through a geometrical analysis, that these type IIB backgrounds were also exact to all orders in α' . This method avoids the complications arising from the worldsheet couplings of the R-R fields. In this article, we extend our analysis to a more general class of K-models with chiral couplings on the worldsheet[15]. On compactification to lower dimensions, these couplings lead to background gauge fields. The bosonic and the heterotic versions of these K-models describe strings propagating in a uniform magnetic field background [20] as one of the special case. These can be formulated as exact conformal field theories and illustrates a phase transition at some critical value of the magnetic field where infinite number of states becomes tachyonic. In lower dimensions they also describe charged black holes through the Kaluza-Klein mechanism [15]. We consider such a K-model in ten dimensions trivially embedded (i.e without R-R fields) in a type IIB background. Using the global $SL(2, R)$ symmetry of the equations of motion, we generate a type IIB background with non-trivial R-R field background. Subsequently, assuming a specific structure for the field strengths, we show that the backgrounds obtained are exact (in α') through a geometrical analysis. In this context, unlike our earlier work, we show that the geometrical considerations based on the covariantly constant null Killing vector may be retranslated in the language of the index structure of the corresponding higher order tensors[17, 19]. It is possible that the present approach is applicable to a wider class of models. This article is divided into four sections. In Section-2 we present a brief review of the backgrounds obtained from K-models

for the bosonic case and show that they are exact. Section-3 deals with K-models embedded in a type IIB theory and their $SL(2, R)$ transformations. In this section we also explicitly prove that these backgrounds are exact in the presence of R-R fields. We present the conclusions in section-4.

2. String Backgrounds from K-Models.

We begin with a description of the string background fields obtained from the K-models. The Lagrangian for the bosonic K-model is given as[15]:

$$\begin{aligned} \mathcal{L} = & 2\partial u \bar{\partial} v + K(u, x) \partial u \bar{\partial} u + 2A_i(u, x) \partial u \bar{\partial} x^i + 2\bar{A}_i(u, x) \bar{\partial} u \partial x^i \\ & + (G_{ij} + B_{ij})(u, x) \partial x^i \partial x^j + R^{(2)}\phi(u, x). \end{aligned} \quad (1)$$

We specialize to the case where $G_{ij} = \delta_{ij}$, $B_{ij} = 0$ and $\phi = \phi(u)$. For this case we have the metric

$$ds^2 = 2dudv + 2A_i^+(u, x)dudx^i + K(u, x)du^2 + dx^i dx_i, \quad (2)$$

and the antisymmetric tensor field:

$$B_{\mu\nu} = \begin{pmatrix} 0 & 1 & A_i^- \\ -1 & 0 & 0 \\ -A_i^- & 0 & 0 \end{pmatrix}. \quad (3)$$

The greek indices (μ, ν) run over $(0 \dots 9)$, (u, v) are the light-cone coordinates and the latin indices $(i, j) = (2 \dots 9)$ run over the transverse space coordinates x^i . We also have,

$$A_i^\pm = A_i \pm \bar{A}_i. \quad (4)$$

The v independence of the metric leads to a killing vector $l^\mu = (0, 1, 0 \dots 0)$. It is possible to express the metric in a compact form in terms of l^μ which eases subsequent computations. Explicitly, we have

$$G_{\mu\nu} = M_{\mu\nu} + K l_\mu l_\nu, \quad (5)$$

where $M_{\mu\nu}$ is a 10×10 symmetric matrix

$$M_{\mu\nu} = \begin{pmatrix} 0 & 1 & A_i^+ \\ 1 & 0 & 0 \\ A_i^+ & 0 & I_8 \end{pmatrix}, \quad (6)$$

and I_8 is a 8×8 unit matrix. The inverse metric is obtained as,

$$G^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -K + A_i^{+2} & -A_i^+ \\ 0 & -A_i^+ & I_8 \end{pmatrix}. \quad (7)$$

The only non-zero connections are Γ_{uu}^i , Γ_{uu}^v , Γ_{ui}^v , Γ_{ui}^j and Γ_{ij}^v . Using these connections, it is easy to show that the null killing vector is covariantly constant. We therefore have $D_\mu l^\nu = 0$ and $D_\mu l_\nu = 0$. The curvature tensor for the backgrounds (7) may be obtained as

$$R_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa}^{(M)} + 2l_{[\lambda}\partial_{\mu]}\partial_{[\nu}Kl_{\kappa]}. \quad (8)$$

Notice that the only non-zero independent components of $R_{\lambda\mu\nu\kappa}$ are R_{uiuj} and R_{uijk} . These are as follows:

$$\begin{aligned} R_{uiuj} &= \frac{1}{2}\partial_i\partial_j K - \frac{1}{2}\partial_u[\partial_i A_j^+ + \partial_j A_i^+] \\ &- \frac{1}{4}G^{mn}(\partial_j A_m^+ - \partial_m A_j^+)(\partial_i A_n^+ - \partial_n A_i^+), \end{aligned} \quad (9)$$

and

$$R_{uijk} = \frac{1}{2}\partial_i[\partial_k A_j^+ - \partial_j A_k^+]. \quad (10)$$

We get the expressions for the corresponding components of the Ricci tensor by appropriate contractions of the Riemann tensor as,

$$R_{uu} = \frac{1}{2}\partial^i\partial_i K - \partial_u\partial^i A_i^+ - \frac{1}{4}(F_{jm} + \bar{F}_{jm})(F^{jm} + \bar{F}^{jm}), \quad (11)$$

$$R_{uk} = -\frac{1}{2}\partial^m(F_{km} + \bar{F}_{km}), \quad (12)$$

where

$$F_{ij} = (\partial_i A_j - \partial_j A_i), \quad \bar{F}_{ij} = (\partial_i \bar{A}_j - \partial_j \bar{A}_i) \quad (13)$$

are the *field strengths* associated with the couplings A and \bar{A} . Next we consider the antisymmetric tensor field strength which is given in the standard form as:

$$H_{\lambda\mu\nu} = (\partial_\lambda B_{\mu\nu} + \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu}). \quad (14)$$

The only non-zero independent component of $H_{\lambda\mu\nu}$ is

$$H_{uij} = -(F_{ij} - \bar{F}_{ij}). \quad (15)$$

We now proceed to obtain the background field equations at tree level. As mentioned earlier, these are provided by the one loop beta functions of the couplings $G_{\mu\nu}$, $B_{\mu\nu}$ and ϕ . Using standard expressions for these equations [8], we obtain three independent equations of motion[15]:

$$\partial^j F_{ij} = 0, \quad \partial^j \bar{F}_{ij} = 0, \quad (16)$$

and

$$-\frac{1}{2}\partial_i\partial^i K + \partial_i\partial^u A_i^+ + F_{ij}\bar{F}^{ij} + 2\partial_u^2\phi = 0. \quad (17)$$

After obtaining the background field equations of the general K-models, we now specialize to the case where the field strengths F_{ij} and \bar{F}_{ij} are constant in the transverse directions but arbitrary functions of u . A subclass of these, namely the ones which are constant in u as well, describe the propagation of closed strings in uniform magnetic field backgrounds[20]. They have been shown to be represented by exact conformal field theory models. The heterotic generalizations of these models have also been found. The chiral couplings in this case are,

$$A_i = -\frac{1}{2}F_{ij}(u)x^j, \quad \bar{A}_i = -\frac{1}{2}\bar{F}_{ij}(u)x^j. \quad (18)$$

As a consequence, we have for the curvature tensor, $R_{uijk} = 0$ and

$$R_{uiuj} = \frac{1}{2}\partial_i\partial_j K - \frac{1}{4}G^{mn}(F_{jm} + \bar{F}_{jm})(F_{in} + \bar{F}_{in}) - \frac{1}{2}\partial_u(F_{ij}(u) + \bar{F}_{ij}(u)). \quad (19)$$

In this case, the first two background field equations in (16) are trivially satisfied. For the metric equation we have,

$$-\frac{1}{2}\partial^i\partial_i K(u, x) + F_{ij}\bar{F}^{ij}(u) + 2\partial_u^2\phi(u) - \frac{1}{2}\partial_u(F_{ii}(u) + \bar{F}_{ii}(u)) = 0. \quad (20)$$

Therefore K must be a quadratic function of the x^i in order to satisfy equation (20).

We now proceed to show that this background is exact to all orders in α' . For this, we first note that that equation (20) is a second rank tensor equation. So we must consider all possible higher order second rank tensor contributions obtained from the background field configuration. The only possible covariant tensor components available for this purpose are, $D_u\phi$, R_{uiuj} and its covariant derivatives with respect to D_u and D_k , H_{uij} and its covariant derivative with respect to D_u . One also has the corresponding contravariant components which are consistent with the form of the metric (2).

We first examine the terms involving a single Riemann tensor with the structure $D^\lambda D^\nu R_{\lambda\mu\nu\kappa}$. An explicit evaluation provides the identities

$$D^u D^u R_{uiuj} = D^i D^j R_{uiju} = D^u D^i R_{ujiu} = D^i D^u R_{iuuj} = 0 \quad etc., \quad (21)$$

which implies

$$D^\lambda D^\nu R_{\lambda\mu\nu\kappa} = 0. \quad (22)$$

It is apparent now that to construct second rank tensors with R_{uiuj} and its derivatives, it is required to contract at least two indices of R with another R or its appropriate derivatives. Potentially non-zero contributions may come from the contractions of covariant indices (u, i) and contravariant indices (v, i) . However this requires a covariant index v or contravariant index u , which are unavailable and contractions on derivatives have been shown to be zero. Hence we conclude that it is impossible to construct non-zero second rank tensors from contractions of $R_{\lambda\mu\nu\kappa}$ and its derivatives [17, 19]. Thus all such higher order contributions are vanishing. Similarly notice that

terms of the form $D_\lambda \phi R^{\lambda\mu\nu\kappa}$ requires contraction of covariant index u and hence it is also zero.

We proceed to consider higher order contributions from the field strength H and its derivatives. Notice that the only non-zero component of H is H_{uij} and the only non-zero covariant derivative is $D_u H_{uij}$. It is obvious that all terms involving only derivatives of H requires contraction of the covariant index u which, as we showed earlier, was not possible. Hence these terms are all identically zero. Higher order contributions of the schematic form $(DR)H$ and $(D\phi)H$ may be proved to be identically zero from similar considerations. It is also possible to show that all scalars constructed from these covariant objects are also vanishing. Hence we conclude that the string background obtained from the K-models are exact to all orders in α' . In the next section we show how these backgrounds may be considered to be trivially embedded in a type IIB string background and generate non-trivial type IIB backgrounds involving R-R fields.

3. Type IIB Backgrounds and Ramond-Ramond Fields.

In this section we now proceed to first show how the backgrounds defined in eqns. (2), (3) and ϕ can be embedded in a type IIB string theory with vanishing R-R fields. We subsequently present the action of the global $SL(2, R)$ transformations on a type IIB background. Utilising these transformations, we then generate a non-trivial type IIB background involving R-R fields. The field content of a type IIB string background consists of the following, the string frame metric $G_{\mu\nu}$, two 3-form field strengths $H_{\lambda\mu\nu}^{(k)}$ where $k = (1, 2)$, two scalars χ and ϕ from the NS-NS and R-R sectors respectively and a 5-form field strength $F_{\lambda\mu\nu\kappa\rho}$. The two scalars χ and ϕ may be combined to form a complex scalar $\lambda = \chi + ie^{-\phi}$. So we may consider the background obtained from the K-model defined by $\phi(u)$, and equations (2), (3) to be a special case of a type IIB background which has $H_{\lambda\mu\nu}^{(2)} = 0$, $\chi = 0$ and $F_5 = 0$. As shown in [6, 7], type IIB strings in $D = 10$ has a global $SL(2, R)$ symmetry at the level of the equations

of motion [7, 21]. This acts on the type IIB background fields as follows :

$$G'_{\mu\nu} = |c\lambda + d| G_{\mu\nu}, \quad (23)$$

$$\lambda' = \frac{a\lambda + b}{c\lambda + d}, \quad (24)$$

and

$$H'^{(k)}_{\lambda\mu\nu} = \Lambda H^{(k)}_{\lambda\mu\nu}, \quad (25)$$

where Λ is an $SL(2, R)$ matrix such that

$$\Lambda = \begin{pmatrix} d & c \\ b & a \end{pmatrix}, \quad (26)$$

with $ad - bc = 1$.

Implementing the transformations outlined in eqns. (23)- (25), we generate a nontrivial type IIB background with R-R fields starting from the trivial type IIB configuration obtained from the K-models, cf eqns. (2), (3) and $\phi(u)$. Explicitly, we have

$$G'_{\mu\nu}(u, x) = f(u) G_{\mu\nu}(u, x), \quad (27)$$

where $f(u) = [d^2 + c^2 e^{-2\phi(u)}]^{\frac{1}{2}}$ and

$$\lambda' = \frac{ia e^{-\phi} + b}{ice^{-\phi} + d}, \quad (28)$$

with $\lambda' = \chi' + ie^{-\phi'}$ and $\lambda = ie^{-\phi}$. We then have the final expressions for the type IIB scalars as:

$$\chi'(u) = \frac{1}{f(u)^2} [db + ac e^{-2\phi}], \quad (29)$$

$$\phi'(u) = \phi(u) + 2 \ln f(u). \quad (30)$$

For the 3-form field strength $H^{(k)}$, $k = 1, 2$ we have

$$H'^{(1)}_{\lambda\mu\nu} = dH^{(1)}_{\lambda\mu\nu}, \quad (31)$$

and

$$H'_{\lambda\mu\nu}{}^{(2)} = bH_{\lambda\mu\nu}{}^{(1)}. \quad (32)$$

The new metric is now given as:

$$ds^2 = 2f(u)dudv + f(u)dx^i dx_i + 2f(u)A_i^+ dudx^i + f(u)K(u, x)du^2, \quad (33)$$

where $K(u, x) = f(u)F(u, x)$. A rescaling $f(u)du = dU$ of the metric leads to the general form

$$ds^2 = -2dUdv + \tilde{f}(U)dx^i dx_i + 2\tilde{A}_i^+(U, x)dUdx^i + \frac{\tilde{K}(U, x)}{\tilde{f}(U)}dU^2. \quad (34)$$

Dropping the tildes and rewriting U as u in (34) we have

$$ds^2 = 2dudv + 2f(u)dx^i dx_i + 2A_i^+(u, x)dudx^i + \hat{K}(u, x)du^2. \quad (35)$$

In subsequent discussions we drop the primes on the non-trivial type IIB background fields generated by the $SL(2, R)$ transformations from the K-model backgrounds. It can be seen from the definitions that \hat{K} in equation (35) is also a quadratic function of x^i 's. This fact becomes important in proving that these backgrounds are all-order solutions of the type IIB equations of motion.

As earlier, the v independence leads to a null Killing vector l^μ . We reexpress the metric in eqn (35) in terms of l^μ as

$$G_{\mu\nu} = M_{\mu\nu} + \hat{K}l_\mu l_\nu. \quad (36)$$

where $M_{\mu\nu}$ is once again a 10×10 matrix given as

$$M_{\mu\nu} = \begin{pmatrix} 0 & 1 & A_i^+ \\ 1 & 0 & 0 \\ A_i^+ & 0 & f(u)I_8 \end{pmatrix} \quad (37)$$

and I_8 is a 8×8 unit matrix. The inverse metric may be easily computed to obtain,

$$G^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -\hat{K} + \frac{A_i^{+2}}{f} & -\frac{A_i^+}{f} \\ 0 & -\frac{A_i^+}{f} & \frac{I_8}{f} \end{pmatrix} \quad (38)$$

Using these, it may be shown that the only non-zero components of the Christoffel connections are, once again, Γ_{uu}^v , Γ_{uu}^i , Γ_{ui}^v , Γ_{ui}^j and Γ_{ij}^v . This leads to the null killing vector being covariantly constant i.e. $D_\mu l^\nu = 0$ and $D_\mu l_\nu = 0$.

We now proceed to compute the Riemann curvature tensor for the metric (35) of the type IIB background generated by us. Employing the closed form expression for the new metric, we once again get the the Riemann tensor to be of the form in equation (8) with $R_{\lambda\mu\nu\kappa}^{(M)}$ now being the Riemann tensor for the metric $M_{\mu\nu}$ in eqn. (37). Once again the only non-zero independent component of the Riemann tensor is R_{uiuj} when we specialize to the background described by eqns. (18). Explicitly, the expression for the Riemann tensor is,

$$\begin{aligned} R_{uiuj} &= \frac{1}{2}\partial_i\partial_j K - \frac{1}{2}\partial_u[\partial^i A_j^+ + \partial_j A_i^+] + \frac{1}{2}\partial_u^2 f \delta_{ij} \\ &- \frac{1}{4}f^{-1}(u)[- \partial_u f \delta_{ik} + (\partial_i A_k^+ - \partial_k A_i^+)] \times [- \partial_u f \delta_{jk} + (\partial_j A_k^+ - \partial_k A_j^+)] \end{aligned} \quad (39)$$

The other background fields are the two antisymmetric tensor field strengths from the NS-NS and the R-R sectors respectively, which are given by $H_{\lambda\mu\nu}^{(k)}$ and $(k = 1, 2)$ in eqns (31), (32). Having obtained the type IIB background with non-trivial R-R fields, described by eqns. (29)-(32) and (35), we now proceed to show that these are exact to all orders (in α'). We once again adopt the geometrical approach [17, 19] outlined in section-2 for this purpose. The background field equations for the type IIB superstring are, to the lowest order, those of N=2 D=10 supergravity in [21]. They are all tensor equations of a definite rank . For the type IIB background under consideration, we have second rank tensor equations for the string form metric $G_{\mu\nu}$ and the antisymmetric tensor field $B_{\mu\nu}$. We also have scalar equations for the NS-NS and the R-R scalars ϕ , χ and a fifth rank completely antisymmetric tensor equation for the five-form field strength $F_{\mu\nu\rho\sigma\kappa}$. The last equation expresses the self duality condition on the five-form field strength.

To study the all order contributions to the background field equations of motion, the possible corrections to all these tensor equations must be considered. Notice

that the contributions from the background gauge fields $B_{\mu\nu}$ and $D_{\mu\nu\rho\sigma}$, appear in the higher order terms as the corresponding gauge invariant field strengths. As a consequence, we need to consider the higher order terms in these equations obtained from the following quantities, $R_{\mu\nu\rho\sigma}$, $H_{\mu\nu\rho}^{(k)}$, $D_\mu\phi$, $D_\mu\chi$, $F_{\mu\nu\rho\sigma\kappa}$ and their covariant derivatives. In our case, we choose F to be zero, which is obtained by setting the four-form field $D_{\mu\nu\rho\sigma} = 0$ in its definition, together with the form of $H^{(k)}$ in eqn. (31), (32).

As in the bosonic case, possible non-zero independent tensor components for the background defined by eqns. (29)-(32) and (35) are $D_u\phi$, $D_u\chi$, R_{uiuj} and its covariant derivatives with respect to D_u and D_j , H_{uij} and its covariant derivatives with respect to D_u and the corresponding covariant components. Notice that the index structures of the appropriate non-zero tensor components are exactly as earlier in the trivial case. The only additions for the field content in the non-trivial type IIB case are the non-zero R-R fields $\chi(u)$ and $H^{(2)}$ as the five-form field strength is zero. Hence similar arguments show that all such higher order contributions as earlier, are vanishing. For the two additional equations also, namely scalar and the five-form R-R fields, similar geometrical arguments show that the higher order contributions vanish. Hence the background field equations which to the lowest order are those of N=2 D=10 supergravity, are exact, to all-orders in α' , also in presence of R-R fields in ten dimensions.

4. Conclusions.

To conclude, we have obtained a class of type IIB superstring backgrounds involving R-R fields from the bosonic K-models embedded in a type IIB background with vanishing R-R fields. The complications involving the sigma model couplings of the R-R fields have been obviated by adopting a purely geometrical approach to compute the higher order terms in the equations of motion. This approach, which is based on the analysis of the tensorial index structure of higher order contributions, seems to be applicable to a wider class of backgrounds than those obtained from the K-models.

We mention *en passant* that our analysis has been restricted to K-model backgrounds with vanishing five form self-dual field strength. Furthermore we have focussed on strictly u dependent antisymmetric tensor field strengths and dilaton. It would be an interesting exercise to obtain type IIB backgrounds from more general K-models and show that they are also exact following the geometrical approach which has been elucidated in this article. In particular, for K-models in the bosonic case, $A = 0$ or $\bar{A} = 0$ conditions provide another class of all-order solutions. It will be interesting to show that they are exact for type IIB as well.

Our results also indicate that type IIB strings in a constant magnetic field background may be formulated as an exact Conformal field theory. It will be interesting to study the phase transition, discussed in [20], for this case. The status of unbroken space-time supersymmetries, like the K-models with trivial embeddings in superstrings, is also of interest to investigate in presence of R-R fields. These will have implications for the present backgrounds to be solutions in the presence of local string-loop corrections as well.

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